



**UNITED STATES DEPARTMENT OF COMMERCE  
Patent and Trademark Office**

Address: COMMISSIONER OF PATENTS AND TRADEMARKS  
Washington, D.C. 20231

APPLICATION NO.	FILING DATE	FIRST NAMED INVENTOR	ATTORNEY DOCKET NO.
09/110,717	07/07/98	MILLS	R 9113-19-C16

IM22/0922  
FARKAS & MANELLI P.L.L.C.  
2000 M STREET, N.W. 7TH FLOOR  
WASHINGTON DC 20036-3307

EXAMINER

KALAFUT, S

ART UNIT

PAPER NUMBER

1745

DATE MAILED:

09/22/00

Please find below and/or attached an Office communication concerning this application or proceeding.

Commissioner of Patents and Trademarks

BEST AVAILABLE COPY

## Office Action Summary

Application No.

09/110717

Applicant(s)

MILLS

Examiner

KALAFET

Group Art Unit

1745

—The MAILING DATE of this communication appears on the cover sheet beneath the correspondence address—

## Period for Reply

3

A SHORTENED STATUTORY PERIOD FOR REPLY IS SET TO EXPIRE \_\_\_\_\_ MONTH(S) FROM THE MAILING DATE OF THIS COMMUNICATION.

- Extensions of time may be available under the provisions of 37 CFR 1.136(a). In no event, however, may a reply be timely filed after SIX (6) MONTHS from the mailing date of this communication.
- If the period for reply specified above is less than thirty (30) days, a reply within the statutory minimum of thirty (30) days will be considered timely.
- If NO period for reply is specified above, such period shall, by default, expire SIX (6) MONTHS from the mailing date of this communication.
- Failure to reply within the set or extended period for reply will, by statute, cause the application to become ABANDONED (35 U.S.C. § 133).

## Status

- ☐ Responsive to communication(s) filed on \_\_\_\_\_.
- ☐ This action is FINAL.
- ☐ Since this application is in condition for allowance except for formal matters, prosecution as to the merits is closed in accordance with the practice under *Ex parte Quayle*, 1935 C.D. 11; 453 O.G. 213.

## Disposition of Claims

- ☒ Claim(s) 1-37 is/are pending in the application.
- ☐ Of the above claim(s) \_\_\_\_\_ is/are withdrawn from consideration.
- ☐ Claim(s) \_\_\_\_\_ is/are allowed.
- ☒ Claim(s) 1-37 is/are rejected.
- ☐ Claim(s) \_\_\_\_\_ is/are objected to.
- ☐ Claim(s) \_\_\_\_\_ are subject to restriction or election requirement.

## Application Papers

- ☒ See the attached Notice of Draftsperson's Patent Drawing Review, PTO-948.
- ☐ The proposed drawing correction, filed on \_\_\_\_\_ is ☐ approved ☐ disapproved.
- ☐ The drawing(s) filed on \_\_\_\_\_ is/are objected to by the Examiner.
- ☐ The specification is objected to by the Examiner.
- ☐ The oath or declaration is objected to by the Examiner.

## Priority under 35 U.S.C. § 119 (a)-(d)

- ☐ Acknowledgment is made of a claim for foreign priority under 35 U.S.C. § 119(a)-(d).
- ☐ All ☐ Some\* ☐ None of the CERTIFIED copies of the priority documents have been received.
- ☐ received in Application No. (Series Code/Serial Number) \_\_\_\_\_.
- ☐ received in this national stage application from the International Bureau (PCT Rule 1.7.2(a)).

\*Certified copies not received: \_\_\_\_\_

## Attachment(s)

- ☒ Information Disclosure Statement(s), PTO-1449, Paper No(s) 2,5,6
- ☒ Notice of Reference(s) Cited, PTO-892
- ☒ Notice of Draftsperson's Patent Drawing Review, PTO-948
- ☐ Interview Summary, PTO-413
- ☐ Notice of Informal Patent Application, PTO-152
- ☐ Other \_\_\_\_\_

Office Action Summary

Art Unit: 1745

1. 35 U.S.C. 101 reads as follows:

Whoever invents or discovers any new and useful process, machine, manufacture, or composition of matter, or any new and useful improvement thereof, may obtain a patent therefor, subject to the conditions and requirements of this title.

2. Claims 1-37 are rejected under 35 U.S.C. 101 because the disclosed invention is inoperative and therefore lacks credible utility. All the claims recite batteries, fuel cells, or electrolytic cells where either a reactant or an electrolyte is a compound including a hydrogen species having an "increased binding energy". This species is also presently called a "hydrino", and is defined in the specification as a "hydrogen atom having the binding energy given in Eq. (2)", which equation recites:

Binding Energy =  $13.6 \text{ eV}/n^2$ , where  $n = 1/p$  and  $p$  is an integer greater than 1.

The "hydrino" would thus be an uncharged hydrogen atom having an "increased binding energy". The present claims also encompass such hydrogen atoms which have been ionized, as well as groups of multiple hydrogen atoms which may be either neutral or ionized, but which in each case exhibit the "increased binding energy". The hydrogen species with a numerically recited binding energy are specific examples of such hydrogen. All of the species presently contemplated, however, are based on applicant's assertion that hydrogen may have energy states which are below the conventionally accepted ground state, as expressed by "Eq. (2)" above.

An asserted utility would not be considered credible where a person of ordinary skill would consider the assertion to be incredible in view of contemporary knowledge and where the evidence offered by an applicant does not counter what contemporary knowledge otherwise

Art Unit: 1745

suggests. According to conventionally accepted scientific principle, the existence of hydrogen with a binding energy corresponding to a value of "n" which is not an integer cannot be mathematically justified. See the attached *Appendix*. According to "Endnote I" of the *Appendix*, Schrödinger's wave equation mandates that the value of "n" must be a positive integer (1, 2, 3, etc.). According to "Endnote V", fractional values for "n" are also impossible in light of the Uncertainty Principle. The fourth full paragraph of page 19-14 of Bethe and Salpeter's *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum Publishing Corporation, New York, 1977), which is presently enclosed, states that the "ground state" of hydrogen has  $n = 1$ . Since applicant's invention is based on a form of hydrogen, which according to conventionally accepted scientific principle cannot exist, the invention would be inoperative and thus lack utility. Thus, the burden is shifted to applicant to provide satisfactory evidence of the operability of the invention, *Newman v. Quigg*, 877 F2d 1575, 11 USPQ2d 1340 (Fed. Cir. 1989).

3. Claims 1-37 are rejected under 35 U.S.C. 112, first paragraph, as containing subject matter which was not described in the specification in such a way as to enable one skilled in the art to which it pertains, or with which it is most nearly connected, to make and/or use the invention. The specification does not enable one of ordinary skill in the art to make or use fuel cells where a reactant is either a hydrogen species having an "increased binding energy", a hydrogen species with a numerically recited binding energy, or a "hydrino", in that undue experimentation would be required.

Art Unit: 1745

Factors to be considered in determining whether a disclosure would require undue experimentation, as set forth by *in re Wands*, 8 USPQ2d 1400, 1404 (Fed. Cir.1988), include:

- (1) the quantity of experimentation necessary,
- (2) the amount of direction of guidance presented in the specification,
- (3) the presence or absence of working examples,
- (4) the nature of the invention,
- (5) the state of the prior art,
- (6) the relative skill of those in the art,
- (7) the predictability or unpredictability of the art,
- (8) the breadth of the claims.

Each of these factors will be addressed as to their relevance to the lack of enablement of the present claims.

(1) The Quantity of Experimentation Necessary

Pages 75-80 of the present specification describe a fuel cell and a battery in which the presently disclosed hydrogen species is an oxidant, which is reacted against a reductant, the latter term being synonymous with the more widely used term “fuel”. Since applicant alleges that the type of hydrogen presently described is new, its electrochemical properties, such as the types of fuels or reductants that it could be reacted against would be presumably unknown. A person skilled in the art would thus have to experiment with a wide variety of reductants to determine which would operated with the presently described hydrogen species in order to produce

Art Unit: 1745

electricity. The specification must teach a person having ordinary skill in the art how to make and use the invention, not merely how the applicant may find out and use the invention himself, *in re Gardner*, 116 USPQ 138, 141 (CCPA 1970).

(2) The Amount of Direction or Guidance Presented in the Specification

Concerning the present claims drawn to a fuel cell, the direction or guidance in the specification is directed to the fuel cell reaction of zinc as a reductant (i.e., fuel) and the presently described hydrogen as the oxidant. Zinc is the only example given in the present disclosure. The guidance present in the specification would thus be directed to one particular fuel cell, the zinc/hydrogen system, whereas the present claims are broad enough to encompass any compound being used as the reductant. Concerning the present claims drawn to a battery, the specification again describes the present hydrogen species as the oxidant, in other words, the cathode active material. As the reductant (the anode active material), there are several examples listed, including water, H<sub>2</sub>, hydroxide, ordinary hydride ion, NH<sub>4</sub>OH and hydrogen halides.

(3) The Presence or Absence of Working Examples

The specification describes, on page 75-80, an example of a fuel cell in which an "increased binding energy" hydrogen species is used as the oxidant, and zinc is the reductant, and an example of a battery with the "increased binding energy" hydrogen species as the oxidant. Several possible reductants are briefly mentioned. The battery may also be "optionally made rechargeable" (page 78, line 23). It is unclear, however, whether applicant has actually made and operated this fuel cell, or any others where the hydrogen species and various reductants are used,

Art Unit: 1745

or which catalyst were used, or could be used. It is also unclear whether applicant has made and operated the variously recited batteries, or batteries where other reductants were used. How the batteries are made rechargeable is not explained. The specification does not show any examples to electrolytic cells including the present hydrogen species within the electrolyte. For these reasons, the present examples are thus not clearly established to be working examples.

#### (4) The Nature of the Invention

The scientific community has held the belief for decades that hydrogen cannot exist below the "ground state", where  $n = 1$ . See the rejection above under §101. The nature of the invention is that it is based on forms of hydrogen which cannot exist under the accepted laws of physics and mathematics. Thus, in order to establish enablement, applicant bears the burden of proving the accepted scientific laws wrong or incomplete, which is not done in the present specification and its examples on pages 75-80. Applicant himself recognizes the unusual nature of his invention. In his book *The Grand Unified Theory Of Classical Quantum Mechanics*, on page 14, applicant states that the theory underlying his invention "predicts the existence of a previously unknown form of matter: hydrogen atoms and molecules having electrons of lower energy than the conventional 'ground' state". The invention is also unusual in that a form of hydrogen is to be used as an oxidant in a fuel cell, reacted against a reductant, whereas hydrogen has normally been used in fuel cells as the reductant itself, reacted against an oxidant such as oxygen.

#### (5) The State of the Prior Art

Art Unit: 1745

There appears to be no prior art showing hydrogen with a binding energy corresponding to “n” being a fraction, below the integer 1, or even any prior art which suggest that this hydrogen could exist in theory, let alone be used as an oxidant in a fuel cell. The state of the fuel cell art would show that hydrogen would normally be a reductant, not an oxidant. There is thus no prior art, in light of which applicant’s disclosure may be read, which would help to enable the existence of compounds having the “increased binding energy” species of hydrogen, or their use as an oxidant in a fuel cell.

(6) Relative Skill of Those in the Art

The most highly skilled people in the art, physicists and chemists familiar with quantum mechanics, would regard the present hydrogen species as something that cannot exist, for reasons set forth in the attached *Appendix*. Any compounds allegedly including such a hydrogen species would also be regarded as things that cannot exist.

(7) The Predictability or Unpredictability of the Art

Because the hydrogen atom with a binding energy corresponding to “n” being less than 1 would be regarded as something that cannot exist, and because the present types of hydrogen species are based on “n” being less than 1, the recognized state of the art would predict against the present compounds which include these forms of hydrogen from ever being formed. At best, since the state of the art does not recognize hydrogen species with an “increased binding energy”, predicting the operation of any fuel cell, battery or electrolytic cell using a compound which includes the present hydrogen species would be extremely difficult, even if these hydrogen species



Art Unit: 1745

were shown to exist. Furthermore, since the very existence of these hydrogen species would be predicted against, the electrochemical properties of such hydrogen species, which could conceivably show them as useful in fuel cells, batteries or electrolytic cells, would be difficult if not impossible to predict.

(8) The Breadth of the Claims

The claims encompass a class of fuel cells which is broader than that shown by lone example to a zinc/hydrogen fuel cell, a class of batteries broader than those shown by the examples to several batteries, and a class of electrolytic cells for which there are no examples.

In conclusion, the present disclosure would require undue experimentation, as seen from a consideration of the above factors. This is mainly because factor (1), the amount of experimentation, and factor (8), the breadth of the claims, are much greater than factor (2), the amount of guidance or direction and the scope of factor (3); and because factors (4) through (7) would show that the invention is based on a type of hydrogen species which cannot exist. The present disclosure is thus considered to be non-enabling.

4. Claims 22-28 are rejected under 35 U.S.C. 112, second paragraph, as being indefinite for failing to particularly point out and distinctly claim the subject matter which applicant regards as the invention. The term "high voltage" would have indefinite scope. No numerical range is disclosed which would define this term.

5. The nonstatutory double patenting rejection is based on a judicially created doctrine grounded in public policy (a policy reflected in the statute) so as to prevent the unjustified or improper timewise extension of the "right to exclude" granted by a patent and to prevent possible

Art Unit: 1745

harassment by multiple assignees. See *In re Goodman*, 11 F.3d 1046, 29 USPQ2d 2010 (Fed. Cir. 1993); *In re Longi*, 759 F.2d 887, 225 USPQ 645 (Fed. Cir. 1985); *In re Van Ornum*, 686 F.2d 937, 214 USPQ 761 (CCPA 1982); *In re Vogel*, 422 F.2d 438, 164 USPQ 619 (CCPA 1970); and, *In re Thorington*, 418 F.2d 528, 163 USPQ 644 (CCPA 1969).

A timely filed terminal disclaimer in compliance with 37 CFR 1.321© may be used to overcome an actual or provisional rejection based on a nonstatutory double patenting ground provided the conflicting application or patent is shown to be commonly owned with this application. See 37 CFR 1.130(b).

Effective January 1, 1994, a registered attorney or agent of record may sign a terminal disclaimer. A terminal disclaimer signed by the assignee must fully comply with 37 CFR 3.73(b).


6. Claims 29-37 are provisionally rejected under the judicially created doctrine of obviousness-type double patenting as being unpatentable over claims 15, 16, 18, 20 and 56 of copending Application No. 09/008,947. Although the conflicting claims are not identical, they are not patentably distinct from each other because the present claims a source of oxidant, which would be the present hydrogen species, called “hydrino” in the copending claims; a cathode which is in communication with that source; an anode; and a salt bridge between the anode and cathode, which each also has its own compartment. The copending claims recite these components along with a vessel, having compartments associated with each electrode. The present claims would thus encompass the copending claims, differing only in that the copending claims require the two compartments to constitute a vessel. The cathode actually being the cathode compartment is recited in present claim 36 and copending claim 56.

This is a provisional obviousness-type double patenting rejection because the conflicting claims have not in fact been patented.

Art Unit: 1745

7. The lengthy specification has not been checked to the extent necessary to determine the presence of all possible minor errors. Applicant's cooperation is requested in correcting any errors of which applicant may become aware in the specification.

8. Any inquiry concerning this communication or earlier communications from the examiner should be directed to examiner Steve Kalafut whose telephone number is (703) 308-0433, and who is also acting as a supervisor. If attempts to reach the examiner are unsuccessful, another acting supervisor, examiner Carol Chaney, may be reached at (703) 305-3777. The Technology Center receptionist may be reached at (703) 308-0661.



STEPHEN KALAFUT  
PRIMARY EXAMINER  
GROUP 1700

sjk

September 19, 2000



Applicant has referred to the book by R. L. Mills entitled *The Grand Unified Theory of Classical Quantum Mechanics* (Blacklight Power Inc., New Jersey, 1999; hereafter, "GUT") which describes the existence of new energy states for the hydrogen atom that are below the conventionally accepted ground state energy. A hydrogen atom in any one of these new energy states is termed a "hydrino." According to equations (I.75a-c) on pages 19-20 of GUT, the general formula representing the energy levels for an electron with a principal quantum number,  $n$ , around the nucleus of the hydrogen atom is:

$$E_n = - (\text{Rydberg constant})/n^2 = - 13.6 \text{ electron volt}/n^2$$

where  $n = 1, 2, 3$ , etc and  $n$  is also  $= 1/2, 1/3, 1/4$ , etc. While the former integer values of  $n$  give energies that are conventionally understood and experimentally verified, the latter fractional values of  $n$  lead to the energies of the electron in a hydrino atom which, according to Mills, represents a new "lower energy hydrogen atom."

A review of some of the main mathematical underpinnings in GUT shows that there is really no proper theoretical basis to assert the existence of the hydrino atom in view of the following discussion.

Nowhere has Mills satisfactorily established that fractional values of  $n$  arise as a natural consequence of a logical and internally consistent mathematical and scientific framework. While GUT bristles with a dense array of mathematical equations, the fractional values of  $n$  are not shown to be the unequivocal end result of Mills' theory. It appears that there is an internal break in logic in the mathematical analysis, with Mills ultimately relying on conclusionary statements, such as, a nonradiative boundary condition and the relationship between the electron and a photon gives transitions in which the electron goes to a "lower" energy nonradiative state with a smaller radius or, alternatively, that an electron can undergo a collision with an "energy hole" which allows the electron to undergo a transition to a lower energy nonradiative state with a smaller radius (pages 16-17 of GUT). In these transitions,

the process involved is called a “shrinkage reaction” yielding a shrunken hydrogen atom accompanied by the release of energy. See pages 16, 17 and 144-146 of GUT.

By way of background, it is noted that there are at least two conventionally recognized approaches to the problem of obtaining the energy levels of the electron in the hydrogen atom. These are:

(a) *Via* a Differential Equation approach formulated as a two-point boundary value problem where boundary conditions at the nucleus and at infinity are imposed on the radial wavefunction of the electron which satisfies a second-order linear differential equation known as Schrödinger’s wave equation. It is to be understood that while the complete wavefunction in spherical polar coordinates is the product of a radial wavefunction and angular wavefunctions, the complete wavefunction for the ground (or lowest energy) state of the hydrogen atom is independent of angular coordinates in view of the spherical symmetry of that state, and is studied only on the basis of the radial wavefunction. Thus, see attached sections 18d-18e and 21b at pages 121-124 and 139 from Pauling and Wilson’s *Introduction to Quantum Mechanics* (Dover Publications, Inc., New York, 1985) and **Endnote 1**.

(b) *Via* an Integral Equation approach wherein the boundary conditions on the radial wavefunction of the electron are “built into” the integral equation itself rather than being imposed on it as in the differential equation formulation. In this approach, upon taking the Fourier transform of the wavefunction, subject to the boundary condition that it satisfies Schrödinger’s equation, an integral equation is obtained. Thus, see attached pages 899-900 from Morse and Feshbach’s *Methods of Theoretical Physics, Part I* (McGraw-Hill Book Company, New York, 1953) and **Endnote 2**.

It is crucial to note that either approach is but a mathematical tool and that, while the integral equation approach may be mathematically more compact and, perhaps, be more convenient for solving certain problems compared to the differential equation approach, the

final results given by either approach must not be mutually contradictory if a scientific theory based on these approaches is to be logical and internally consistent.

From a consideration of Mills' mathematical derivations on pages 4-5 (equations (I.5) to (I.11)) , on pages 32-38 (equations (1.3) to (1.45)) and on pages 136-141 (equations (5.1) to (5.21)) of GUT, it appears that Mills' formulation *may* be an integral equation type of approach. Specifically, the boundary condition "built into" the integral equation is an expression for the current density, and, thus, the charge density of a point charge which satisfies Maxwell's equation for the electric field as given by Haus in a paper, in the *American Journal of Physics*, vol. 54, no. 12, pages 1126-1129 (1986), relating to the absence of radiation from a point charge moving at constant velocity. See page 3 of GUT. While, Haus' paper is not the focus of discussion here, it is apparent that the use of a Dirac delta function,  $\delta(\mathbf{r}-\mathbf{r}_0)$ , to represent the electron charge density on page 4 of GUT may be an unphysical assumption in that, whereas the electron charge density is an "observable" that is ultimately measurable, the delta function, which purports to represent it, is not, in and of itself, a function in the usual mathematical sense of the term and is physically meaningful only under an integral sign.

More specifically, it appears that Mills' integral equation approach utilizes the technique of the "Green's function." In the theory of integral equations, the Green's function is a function that satisfies a differential equation involving a Dirac delta function type of point source. A connection between the Green's function and the wavefunction is established by requiring the former to satisfy boundary conditions corresponding to those satisfied by the latter. Interpreting Mills' equations as best as one can, it is possible, though by no means certain, that Mills achieves such a connection by requiring the Green's function to satisfy boundary conditions imposed on the charge density function in Mills' equation (1.1) on page 31 of GUT. The final step in the integral equation approach is to generate an integral

equation involving an integral taken over the Green's function. The solution of that equation would yield the wavefunction of the electron and, from that, leads to the energy levels of the electron in the hydrogen atom. See attached pages 808, 902 and 903 from Morse and Feshbach *op. cit.* and Endnote 3. It is observed that the legitimate use of a Green's function which satisfies an equation involving a Dirac delta function type of "point source" and appears, ultimately under an integral sign as the kernel of an integral equation, does not justify Mills' representation of the electron charge density, which is a "smeared out" charge distribution, as a Dirac delta function as discussed previously. Mills' lack of consistency in using properly subscripted variables as well as the absence of a logical flow in the mathematical derivations, prevents one from properly assessing the kind of approach taken in GUT.

In any event, at least some problematical issues are seen in the Mills' treatment, *viz.*, (i) it is not explained as to why it is physically meaningful to utilize Haus' boundary condition for a classical point charge moving in free space in order to obtain the energy levels of the electron in a quantized system such as the hydrogen atom where the electron moves in a confined space due to its attractive coulombic interaction with the positively charged nucleus, and, (ii) there is no explanation for the catastrophic collapse of the electron into the nucleus as  $n \rightarrow \infty$  in the fractional quantum number series,  $1/n$ , i.e. the hydrino atom implodes and ceases to exist. See pages 144-146 of GUT. The end result of Mills' integral equation approach, if such it is, fails to bear out his assertion that  $n$  must unequivocally have fractional values. In essence, it appears that the condition that  $n$  have fractional values (see equations (I.75c) and (2.2) on pages 20 and 81 of GUT) is but an *ad hoc* statement that does not logically flow from Mills' derivation of the equation for the energy levels of the electron in the hydrogen atom and it may even represent a type of forced parameterization scheme

deliberately structured to produce a desired outcome contrary to the logical flow of its mathematics or, even, common sense.

Hence, it appears that Mills' theory remains essentially unproven as discussed above and does not constitute a proper basis to demonstrate the existence, at least on theoretical grounds alone, of the so-called hydrino atom.

Furthermore, Mills' theory does not show that the conventional quantum mechanical treatment of the hydrogen atom is theoretically or experimentally flawed. Any attempt to establish a new result for the hydrogen atom that is presently unknown to quantum mechanics must cross a rather steep threshold of scientific credibility. See the attached page 2 from Bethe and Salpeter's *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum Publishing Corporation, New York, 1977 and **Endnote 4**.

Among the many problems solved by quantum mechanics, the hydrogen atom, along with the linear harmonic oscillator and the particle-in-a-box, is one of the few scientific problems that has received extensive theoretical and experimental treatment over many years since the first decade of the twentieth century. For a complete treatment of the hydrogen atom problem see the attachment from pages 19-1 to 19-18 of Feynman's *Lectures in Physics*, vol. III, *Quantum Mechanics* (Addison-Wesley Publishing Co., Reading, Mass., 1965). The results obtained from at least one type of standard procedure for solving the radial Schrödinger equation using a power series expansion for the wavefunction of the electron inescapably lead to the conclusion that only positive integer values for  $n$  are permissible (as explained previously in **Endnote 1**). See attached pages 1-9 and 2-6 from Feynman *op. cit.* and **Endnote 5**. In other words, conventional theory and experiment forbid hydrino atoms.



### Endnote 1

Schrödinger's wave equation for the radial wavefunction,  $S(\rho)$ , is:

$$(1/\rho^2)(d/d\rho)(\rho^2 dS/d\rho) + \{-1/4 - l(l+1)/\rho^2 + \lambda/\rho\} S = 0$$

where  $\rho$  is proportional to the radial coordinate in the spherical polar coordinate system with  $0 \leq \rho \leq \infty$ ,  $l$  is the orbital angular momentum quantum number and  $\lambda$  is proportional to negative (i.e. bound) energy values. The boundary conditions are that far from the nucleus of the hydrogen atom ( $\rho \rightarrow \infty$ ) the radial wavefunction becomes negligible i.e.  $S \rightarrow 0$ , and, at the nucleus of the atom ( $\rho = 0$ ), noting that  $S$  is expressible as  $e^{-\rho/2} \rho^s L(\rho)$  where  $L = \sum_v a_v \rho^v$  is an infinite power series in  $\rho$ , substitution of the expression for  $S$  into the radial wavefunction equation results in the choice of  $s = +l$  (which is a positive integer) as the only choice that will permit to be  $S$  be an acceptable wavefunction, which in turn yields the boundary condition that  $S$  has a finite value at the nucleus. Note that despite the finite value of the radial wavefunction at the nucleus, the probability of finding the electron at the nucleus,  $\rho = 0$ , of the hydrogen atom in its normal ground state is proportional to  $4\pi\rho^2 S^2$  which, of course, is zero. Upon substituting the cited expression for  $S$  into the radial wavefunction equation, recursion relations between  $a_v$  for various values of  $v$  are obtained. The recursion relations contain the principal quantum number  $n$  appearing as a multiplicative coefficient of  $a_v$ . Since  $S$  must have a proper asymptotic behavior as  $\rho \rightarrow \infty$ , this requires that the infinite power series be terminated after a finite number of terms which in turn, after some algebra, leads to the result that  $n$  must be a positive integer having the values 1, 2, 3, etc.. See equations (18.29) to (18.39) and Figure 21-1 at pages 121-124 and 140 in Pauling and Wilson.

### Endnote 2

Substitution of the Fourier transform of the wavefunction,  $\psi(\mathbf{r})$ , viz.

$$\psi(\mathbf{r}) = (1/h)^{3/2} \int_{-\infty}^{\infty} \varphi(\mathbf{p}) e^{(2\pi i/h)\mathbf{p}\cdot\mathbf{r}} d\mathbf{p},$$

where  $h$  is Planck's constant and  $\mathbf{p}$  and  $\mathbf{r}$  are momentum and spatial coordinate vectors, respectively, into the Schrödinger equation in the differential form

$$\nabla^2 \psi + (2m/h^2) \{E - V[\mathbf{r}, (h/2\pi i)\nabla]\} \psi = 0,$$

where  $\nabla^2$ ,  $E$  and  $V$  are the Laplacian operator, total and potential energies, respectively, followed by multiplication through by  $(1/h)^{3/2} e^{(-2\pi i/h)\mathbf{q}\cdot\mathbf{r}}$  and an integration over  $\mathbf{r}$  yields the desired integral equation

$$(q^2/2m)\varphi(\mathbf{q}) + \int_{-\infty}^{\infty} \varphi(\mathbf{p}) V(\mathbf{p}-\mathbf{q}, \mathbf{p}) d\mathbf{p} = E\varphi(\mathbf{q})$$

where

$$V(\mathbf{p}-\mathbf{q}, \mathbf{p}) = (1/h)^{3/2} \int_{-\infty}^{\infty} e^{(2\pi i/h)(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}} V(\mathbf{r}, \mathbf{p}) d\mathbf{r}$$

with  $\mathbf{q}$  being a momentum vector.

See equation (8.1.4) at page 900 in Morse and Feshbach.

### Endnote 3

To illustrate a method of obtaining a solution for the wavefunction,  $\psi$ , by the technique of Green's functions consider the Schrödinger equation for  $\psi$  written as:

$$[\nabla^2 + k^2]\psi = U\psi$$

where  $k^2 = (8\pi^2 m/h^2)E$  and  $U = (8\pi^2 m/h^2)V$  with  $E$  and  $U$  being the total and potential energies, respectively. A Green's function,  $G_k(\mathbf{r}|\mathbf{r}_0)$ , is introduced which satisfies

$$[\nabla^2 + k^2]G_k(\mathbf{r}|\mathbf{r}_0) = -4\pi\delta(\mathbf{r}-\mathbf{r}_0),$$

where  $\delta(\mathbf{r}-\mathbf{r}_0)$  is a Dirac delta function representing a "point source" at  $\mathbf{r}_0$ . The Green's function can be thought of as representing an effect at  $\mathbf{r}$  caused by a point source at  $\mathbf{r}_0$ . The

boundary conditions on  $G_k(\mathbf{r}|\mathbf{r}_0)$  are chosen to be the same as those corresponding to the boundary conditions on the wavefunction  $\psi$ . Then, by the theory of integral equations, a solution to the Schrödinger equation is:

$$\psi(\mathbf{r}) = -(1/4\pi) \int G_k(\mathbf{r}|\mathbf{r}_0) U(\mathbf{r}_0) \psi(\mathbf{r}_0) d\mathbf{r}_0.$$

See pages 808, 902 and 903 in Morse and Feshbach.

#### Endnote 4

Regarding the study of the hydrogen atom, note the following quotation from page 2 of Bethe and Salpeter's classic text entitled *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum Publishing Corporation, New York, 1977):

"One of the simplest, and most completely treated, fields of application of quantum mechanics is the theory of atoms with one or two electrons. For hydrogen and the analogous ions  $\text{He}^+$ ,  $\text{Li}^{++}$ , etc., the calculations can be performed exactly, both in Schrödinger's nonrelativistic wave mechanics and in Dirac's relativistic theory of the electron. More specifically, the calculations are exact for a single electron in a fixed Coulomb potential. Hydrogen-like atoms thus furnish an excellent way of testing the validity of quantum mechanics. For such atoms the correction terms due to the motion and structure of atomic nuclei and due to quantum electrodynamic effects are small and can be calculated with high accuracy. Since the energy levels of hydrogen and similar atoms can be investigated experimentally to an astounding degree of accuracy, some accurate tests of the validity of quantum electrodynamics are also possible."

### Endnote 5

It is noteworthy that this position is also supported by a different line of argument that is independent of the solution to Schrödinger's equation. Thus, fractional values for the principal quantum number  $n$  would bring the electron much closer to the nucleus of the hydrogen atom than is permitted by Heisenberg's Uncertainty Principle. Feynman has presented a mathematically simple argument, in his "Lectures in Physics," vol. III, page 2-6, to show that the size of the hydrogen atom i.e. when  $n$  is 1 (rather than, say,  $1/2$ ) is perfectly consistent with the Uncertainty Principle. This argument goes as follows: from the Uncertainty Principle, if the electron is at a distance  $a$  from the hydrogen nucleus, then the product of its momentum and  $a$  must be of the order of Planck's constant. Now the total energy of the electron is the sum of its kinetic and potential energies. Noting that the kinetic energy can be expressed in terms of the square of the momentum, upon invoking the value of the momentum from the Uncertainty Principle and minimizing the total energy in order to obtain the lowest energy level of the electron, one immediately obtains the standard result for the lowest energy level of the electron in the hydrogen atom which is consistent with  $n$  being 1 and no lower than 1. Since, according to Feynman, "no one has ever found (or even thought of) a way around the Uncertainty Principle ... so we must assume it describes a basic characteristic of nature," (page 1-9 in Feynman) it appears that Mills' fractional value for  $n$  is impermissible in light of the inviolability of the Uncertainty Principle.

---